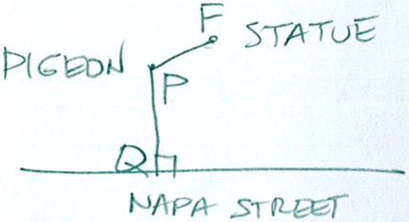
Sonoma Plaza is a square with a statue in the middle. Napa Street is the road that marks the south side of the square. **SCORE: _____ / 10 PTS**
A pigeon is walking around the plaza so that it is always five times as far from Napa Street as it is from the statue. What is the shape of the pigeon's path ? **<u>Draw a diagram and write algebraic equations involving distances to justify your answer.</u>**



3

$PQ = 5PF$

$\frac{1}{5} = \frac{PF}{PQ} = e \longrightarrow$ ELLIPSE

4 3

AJ and BJ were working on their polar graphing partner quiz. **SCORE: _____ / 40 PTS**

On the question about the polar equation $r = \sqrt{3} + 2\sin 3\theta$, they determined correctly that the symmetry tests $(-r, \pi - \theta)$, $(-r, \theta)$, $(-r, -\theta)$ and $(r, -\theta)$ do **NOT** indicate that the graph is symmetric.

AXIS POLE $\theta = \frac{\pi}{2}$ AXIS

[a] **Using their results, along with the tests and shortcuts shown in lecture**, test if the graph is symmetric over the pole, the polar axis and/or $\theta = \frac{\pi}{2}$. State your conclusions in the table. **NOTE: Run as FEW tests as needed to prove your answers are correct.**

POLE: $r = \sqrt{3} + 2\sin 3(\pi + \theta)$ 3

3 $r = \sqrt{3} + 2\sin(3\pi + 3\theta)$

$r = \sqrt{3} + 2(\sin 3\pi \cos 3\theta + \cos 3\pi \sin 3\theta)$ (0, −1)

3 $r = \sqrt{3} - 2\sin 3\theta$ ×

$\theta = \frac{\pi}{2}$: $r = \sqrt{3} + 2\sin 3(\pi - \theta)$ 3

SIGN CHANGE FROM PRIOR TEST

2 $r = \sqrt{3} + 2\sin 3\theta$ ✓

Type of symmetry	Conclusion
Over the pole	2 NO CONCLUSION
Over the polar axis	2 NO CONCLUSION
Over $\theta = \frac{\pi}{2}$	2 SYMMETRIC

[b] Based on the results of part [a], what is the minimum interval of the graph you need to plot (before using reflections to draw the rest of the graph) ?

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ 5

[c] Find all angles **algebraically** in the minimum interval in part [b] at which the graph goes through the pole.

$r = \sqrt{3} + 2\sin 3\theta = 0$ $\qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

3 $\sin 3\theta = -\frac{\sqrt{3}}{2}$ $\qquad -\frac{3\pi}{2} \le 3\theta \le \frac{3\pi}{2}$

9 $3\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{4\pi}{3}$

3 $\theta = -\frac{2\pi}{9}, -\frac{\pi}{9}, \frac{4\pi}{9}$

Name the shape of the graphs of the following polar equations.
If the graph is a rose curve, state the number of petals.

SCORE: ____ / 20 PTS

$r = \frac{5}{1 - \frac{3}{2}\cos\theta}$

[a] $r = \dfrac{10}{2 - 3\cos\theta}$ HYPERBOLA 3

$|-4| < |7| < 2|-4|$

[b] $r = 7 - 4\cos\theta$ LIMAÇON WITH DIMPLE 3

[c] $r = 8\sin 9\theta$ ROSE CURVE 9 PETALS 5

[d] $r = \pi$ CIRCLE 3

$|4| = |-4|$

[e] $r = 4 - 4\sin\theta$ CARDIOID 3

$r = \frac{\frac{10}{3}}{1 + \frac{2}{3}\sin\theta}$

[f] $r = \dfrac{10}{3 + 2\sin\theta}$ ELLIPSE 3

Rewrite $\operatorname{csch}(-4\ln x)$ in terms of exponential functions and simplify. **SCORE: _____ / 10 PTS**

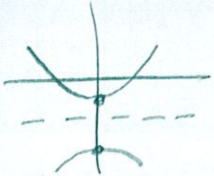
$$\boxed{\frac{2}{e^{-4\ln x}-e^{4\ln x}}} = \boxed{\frac{2}{x^{-4}-x^{4}}} \cdot \frac{x^4}{x^4} = \boxed{\frac{2x^4}{1-x^8}}$$

3 4 3

A hyperbola has a focus at the pole and vertices with **rectangular** co-ordinates $(0, -1)$ and $(0, -9)$.

SCORE: _____ / 20 PTS

[a] Find polar co-ordinates for the vertices, using positive values of r and θ.



$\left(1, \frac{3\pi}{2}\right)$ $\left(9, \frac{3\pi}{2}\right)$ 2

USE $\left(-9, \frac{\pi}{2}\right)$

[b] Find the **polar** equation of the hyperbola.

$r = \dfrac{ep}{1 - e\sin\theta}$ 4

$1 = \dfrac{ep}{1+e}$ 2 $\quad -9 = \dfrac{ep}{1-e}$ 2

$ep = 1 + e \qquad ep = -9 + 9e$

$1 + e = -9 + 9e$ 2

$e = \frac{5}{4}$ 2

$\frac{5}{4}p = 1 + \frac{5}{4} \longrightarrow p = \frac{9}{5}$ 2

2 $r = \dfrac{\frac{5}{4} \cdot \frac{9}{5}}{1 - \frac{5}{4}\sin\theta} \cdot \dfrac{4}{4}$

2 $r = \dfrac{9}{4 - 5\sin\theta}$

CJ is standing 36 feet from DJ, who is 6 feet tall. **SCORE: _____ / 30 PTS**

CJ throws a football at 30 feet per second in DJ's direction, at an angle of 36.87° with the horizontal, from an initial height of 5 feet.

NOTE: $\sin 36.87^\circ = \frac{3}{5}$ **and** $\cos 36.87^\circ = \frac{4}{5}$

ALL ITEMS
3 POINTS

[a] Write parametric equations for the position of the football.

$x = (30\cos 36.87^\circ)t = 30 \cdot \frac{4}{5}t = 24t$

$y = 5 + (30\sin 36.87^\circ)t - 16t^2 = 5 + 30 \cdot \frac{3}{5}t - 16t^2$

$= 5 + 18t - 16t^2$

[b] Does the football hit DJ, go over DJ's head, or hit the ground before reaching DJ ?
Your solution must use the answer to part [a].

$x = 24t = 36 \rightarrow t = \frac{3}{2}$

$y = 5 + 18 \cdot \frac{3}{2} - 16 \cdot \left(\frac{3}{2}\right)^2$

$= 5 + 27 - 36$

$= -4 < 0 \longrightarrow$ HIT THE GROUND

Find the logarithmic formula for $\sinh^{-1} x$ by solving $x = \sinh y$ for y using the exponential definition and an algebraic substitution $z = e^y$.

SCORE: _____ / 20 PTS

$$x = \frac{e^y - e^{-y}}{2} = \frac{z - \frac{1}{z}}{2} \cdot \frac{z}{z} = \frac{z^2 - 1}{2z}$$

$$z^2 - 2xz - 1 = 0$$

$$z = \frac{2x \pm \sqrt{4x^2 + 4}}{2} = x \pm \sqrt{x^2 + 1}$$

ALL ITEMS

$2\frac{1}{2}$ POINTS

$e^y = z = x + \sqrt{x^2 + 1}$ SINCE $e^y > 0$ BUT $x - \sqrt{x^2 + 1} < 0$

$$y = \ln\left(x + \sqrt{x^2 + 1}\right) = \sinh^{-1} x$$